Supplementary Materials

APPENDIX A

PHYSICAL INTERPRETATION OF FUEL-RATE-SPEED FUNCTION

A truck running on a road with grade/slope $\theta$ (positive if moving up and negative if moving down) faces three resistances: aerodynamic (air) resistance, rolling resistance and grade resistance [62]. The air resistance is the friction of air, which is modeled as

$$ F_a(v) = \frac{1}{2} \rho A_f c_d v^2, $$

where $\rho$ is the air density and $A_f$ the frontal area of the truck and $c_d$ is drag coefficient of the truck (see Tab. III for $c_d$ and $A_f$) and $v$ is the speed of the truck. The rolling resistance is the friction between the tires and the ground, which is modeled as

$$ F_r = c_r mg \cos \theta, $$

where $c_r$ is the coefficient of rolling resistance (friction coefficient) between the tires and the ground, $m$ is the truck mass and $g$ is gravitational acceleration. The grade resistance is the force of the gravity on the opposite direction of truck movement, i.e.,

$$ F_g = mg \sin \theta. $$

Then the tractive force is

$$ F_t(v) = F_a(v) + F_r + F_g, $$

which yields to the power consumption

$$ P_f(v) = F_t(v) \cdot v = \frac{1}{2} \rho A_f c_d v^3 + (c_r \cos \theta + \sin \theta) mg v. $$

We can regard $P_f(v)$ as the power demand to move the truck on the road with constant speed $v$. To provision such power demand, the internal combustion engine (ICE) needs to convert fuel into mechanical energy. There are a substantial number of models for ICE [49]. For the purpose of this physical interpretation, we use the following relationship (see [49, Equation 10]),

$$ P_f = f(v) \cdot \text{LHV} \cdot \eta, $$

where $f(v)$ is the fuel rate consumption (unit: gallon per hour), LHV is the lower heating value of the fuel (unit: KJ per gallon), and $\eta$ is the fuel efficiency\(^8\). Eq. (23) gives the fuel-rate-speed function $f(v)$ as follows,

$$ f(v) = \frac{P_f}{\text{LHV} \cdot \eta} = \frac{1}{2} \rho A_f c_d v^3 + (c_r \cos \theta + \sin \theta) mg v. $$

which shows that the fuel-rate-speed function is polynomial with speed $v$ and also strictly convex.

Therefore, such physical interpretation justifies our assumption for the fuel-rate-speed function in Sec. II-A.

\(^8\)The unit of power demand $P_f$ would be KW. We can appropriately make all units consistent.

APPENDIX B

PROOF OF LEMMA 1

We can prove this lemma by using the continuous Jensen’s inequality. For any speed profile $v : [0, t_e] \rightarrow \mathbb{R}^+$ over road/edge $e$, the incurred fuel consumption is $\int_0^{t_e} f_e(v(t)) dt$, and the travelled distance is $\int_0^{t_e} v(t) dt$. As we require that the truck must pass edge $e$ with exactly $t_e$ hours, we must have

$$ \int_0^{t_e} v(t) dt = D_e. $$

Since $f_e(\cdot)$ is convex, according to the continuous Jensen’s inequality [63, Ch. 12.411], we have

$$ \frac{\int_0^{t_e} f_e(v(t)) dt}{t_e} \geq f_e\left( \frac{\int_0^{t_e} v(t) dt}{t_e} \right) = f_e\left( \frac{D_e}{t_e} \right), $$

which means

$$ \int_0^{t_e} f_e(v(t)) dt \geq t_e f_e\left( \frac{D_e}{t_e} \right), $$

with equality when $v(t) = \frac{D_e}{t_e}$ for all $t \in [0, t_e]$.

The proof is completed.

APPENDIX C

PROOF OF LEMMA 2

Since the fuel-rate-speed function $f_e(v)$ is a polynomial function (and thus twice differentiable) with respect to $v$, we can thus obtain the first and second-order derivative of $c_e(t_e) = t_e f_e\left( \frac{D_e}{t_e} \right)$ with respect to $t_e$, i.e.,

$$ c'_e(t_e) = f'_e\left( \frac{D_e}{t_e} \right) \frac{D_e}{t_e} - \frac{D_e}{t_e} f''_e\left( \frac{D_e}{t_e} \right), $$

and

$$ c''_e(t_e) = f''_e\left( \frac{D_e}{t_e} \right) \left( \frac{D_e}{t_e} \right)^2 - \left( \frac{D_e}{t_e} f'_e\left( \frac{D_e}{t_e} \right) + \frac{D_e}{t_e} f''_e\left( \frac{D_e}{t_e} \right) \left( - \frac{D_e}{t_e} \right) \right) $$

$$ = \frac{D_e^2}{t_e^3} f''_e\left( \frac{D_e}{t_e} \right). $$

Since $f_e(\cdot)$ is strictly convex over the speed limit region, we have $f''_e\left( \frac{D_e}{t_e} \right) > 0$ and thus we conclude that

$$ c''_e(t_e) > 0, $$

which proves that $c'_e(\cdot)$ is strictly convex with respect to $t_e$ over $[t_e^{lb}, t_e^{ub}]$.

For the second part of this lemma, we first observe that $c'_e(t_e)$ is a differentiable (and thus continuous) and strictly increasing function. Thus we will consider the following three cases.

Case 1 $0 \leq c'_e(t_e^{lb})$: In this case, we know that $c'_e(t_e)$ is strictly increasing over $[t_e^{lb}, t_e^{ub}]$ and we can set $t_e = t_e^{lb}$.

Case 2 $0 \in (c'_e(t_e^{lb}), c'_e(t_e^{ub}))$: In this case, we can find a $t_e \in (c'_e(t_e^{lb}), c'_e(t_e^{ub}))$ such that $c''_e(t_e) = 0$ due to the continuity of $c'_e(t_e)$.

Case 3 $0 \geq c'_e(t_e^{ub})$: In this case, we know that $c'_e(t_e)$ is strictly decreasing over $[t_e^{lb}, t_e^{ub}]$ and we can set $t_e = t_e^{ub}$.
In all three cases, we obtain that $c_e(t_e)$ is first strictly decreasing over $[\hat{t}_e^l, \hat{t}_e]$ and then strictly increasing over $[t_e^u, t_e^r]$. Note that $t_e$ could be on the boundary of $[t_e^u, t_e^r]$, as shown in Case 1 and Case 3.

The proof is completed.

**APPENDIX D**

**PROOF OF LEMMA 3**

First, since $p$ and $t_p$ is a feasible solution to PASO, we have $\text{OPT} \leq \tilde{c}(p, t_p)$.

Second, since Algorithm 2 returns in line 13, the path cost will be no greater than some $c \leq N$, thus we have

$$\hat{c}(p, t_p) \leq \sum_{e \in p} \tilde{c}_e(t_e) = \sum_{e \in p} \min\{\left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1, N + 1\} \leq N,$$

which clearly implies that

$$\hat{c}_e(t_e) = \left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1, \forall e \in p.$$

Then we have

$$\hat{c}(p, t_p) = \sum_{e \in p} \hat{c}_e(t_e) = \sum_{e \in p} \left[ \frac{c_e(t_e)}{V} \right] + 1 \geq \sum_{e \in p} \hat{c}_e(t_e) = \frac{c(p, t_p)}{V},$$

which yields to

$$c(p, t_p) \leq \hat{c}(p, t_p)V \leq NV = \left(\left\lfloor \frac{U}{V} \right\rfloor + n + 1\right)V \leq \left(\frac{U}{V} + n + 1\right)V = U + (n + 1)V = U + L\delta.$$

The proof is completed.

**APPENDIX E**

**PROOF OF LEMMA 4**

For PASO, let us denote $(p^*, t^*_p)$ as an optimal solution. Namely, $p^*$ is an optimal path and $t^*_p$ is the corresponding optimal travel time set. For each edge $e \in p^*$, we must have

$$\min\{\left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1, N + 1\} = \left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1.$$

Suppose not. Then

$$\left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1 > N + 1,$$

which means

$$c_e(t_e) \geq V\left\lfloor \frac{c_e(t_e)}{V} \right\rfloor > V N = V\left(\left\lfloor \frac{U}{V} \right\rfloor + n + 1\right) > U \geq \text{OPT}.$$

This is a contradiction to $c_e(t_e) \leq \sum_{e \in p^*} c_e(t_e) = \text{OPT}$.

Then we have

$$\hat{c}(p^*, t^*_p) = \sum_{e \in p^*} \hat{c}_e(t_e) = \sum_{e \in p^*} \min\{\left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1, N + 1\}$$

$$= \sum_{e \in p^*} \left[ \frac{c_e(t_e)}{V} \right] + 1 \leq \sum_{e \in p^*} \left[ \frac{c_e(t_e)}{V} \right] + 1$$

$$\leq \frac{\text{OPT}}{V} + n \leq \frac{U}{V} + n \leq (\left\lfloor \frac{U}{V} \right\rfloor + 1) + n = N.$$

Here is a critical step which is different from Lemma 3 in [51] for RSP problem. For each edge $e \in p^*$, $t_e$ may not be a representative point in vector $\tau_e$. However, we can consider the representative point $\hat{t}_e = \tau_e$ where $\hat{t}_e \triangleq \hat{c}_e(t_e)$, which incurs the same fuel cost, i.e., $c_e(t_e) = \hat{c}_e(t_e)$. Clearly, we also have $\hat{c}(p^*, t^*_p) \leq N$ and $t_e \leq \hat{t}_e$ where $t^*_p \triangleq \{\hat{t}_e : e \in p^*\}$.

Therefore path $p^*$ and travel time $t^*_p$ must be examined by Algorithm 2, which completes the proof of the first part, i.e., Algorithm 2 must return a feasible path $p$ and travel time $t_p$. Moreover, we have

$$\hat{c}(p, t_p) \leq \hat{c}(p^*, t^*_p) = \hat{c}(p^*, t^*_p).$$

From (31), we first note that

$$\hat{c}(p^*, t^*_p) \leq \frac{\text{OPT}}{V} + n.$$

Second, since Algorithm 2 returns in line 13, we must have

$$\hat{c}(p, t_p) = \sum_{e \in p} \hat{c}_e(t_e) = \sum_{e \in p} \min\{\left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1, N + 1\} \leq N,$$

which clearly implies that

$$\hat{c}_e(t_e) = \left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1, \forall e \in p.$$

We then note that

$$\hat{c}(p, t_p) = \sum_{e \in p} c_e(t_e) = \sum_{e \in p} \min\{\left\lfloor \frac{c_e(t_e)}{V} \right\rfloor + 1, N + 1\}$$

$$= \sum_{e \in p} \left[ \frac{c_e(t_e)}{V} \right] + 1 \geq \sum_{e \in p} \hat{c}_e(t_e) = \frac{c(p, t_p)}{V},$$

Inserting inequalities (33) and (34) into (32), we obtain

$$c(p, t_p) \leq \frac{\text{OPT}}{V} + n,$$

which means

$$c(p, t_p) \leq \text{OPT} + nV \leq \text{OPT} + L\delta.$$

The proof is completed.

**APPENDIX F**

**PROOF OF THEOREM 2**

The first part of this theorem directly follow the analysis of Steps 1-3 in Sec. III-C. Namely, Algorithm 3 returns a $(1 + \epsilon)$-approximate solution for PASO in time

$$O((mn \log \xi + mn^2) \log \log \frac{\text{UB}}{\text{LB}} + \frac{mn \log \xi}{\epsilon} + \frac{mn^2}{\epsilon^2}).$$

Now we prove the second part of this theorem. Namely, if we use $\text{LB} = C_{\text{lb}}$ and $\text{UB} = nC_{\text{ub}}$, where $C_{\text{lb}} \triangleq \min_{e \in E} c_e(t_e^{lb})$ and $C_{\text{ub}} \triangleq \max_{e \in E} c_e(t_e^{ub})$, Algorithm 3 has time complexity polynomial in the input size of the problem PASO and therefore is an FPTAS. According to (35), we only need to show log log $\frac{\text{UB}}{\text{LB}} = \log \log \frac{nC_{\text{ub}}}{C_{\text{lb}}}$ is polynomial in the input size.
Suppose that $C_{ub} \triangleq \max_{e \in \mathcal{E}} C_e(t_{eb}^l) = c_{e_1}(t_{eb}^l)$. For edge $e_1$, we should input all its properties, i.e., \{ $D_{e_1}$, $R_{e_1}^{lb}$, $R_{e_1}^{ub}$, $f_{e_1}$ \} where $f_{e_1}$ is a polynomial function. Suppose that

$$f_{e_1}(x) = a_1 x^{k_1} + a_2 x^{k_2} + \cdots + a_q x^{k_q}.$$ 

Then to input fuel-rate-speed function $f_{e_1}$, we only need to input $a_1, k_1, a_2, k_2, \cdots, a_q, k_q$. Therefore, for edge $e_1$, we should input the following real numbers,

\begin{equation*}
\{ D_{e_1}, R_{e_1}^{lb}, R_{e_1}^{ub}, a_1, k_1, a_2, k_2, \cdots, a_q, k_q \}.
\end{equation*}

The input size for edge $e_1$ is

$$I_{e_1} = \log \left( \frac{D_{e_1} + R_{e_1}^{lb} + R_{e_1}^{ub} + a_1 + k_1 + a_2 + k_2 + \cdots + a_q + k_q}{\epsilon_{ps}} \right).$$

where $\epsilon_{ps} \ll 1$ is the machine epsilon, i.e., the maximum relative error of for rounding a real number to the nearest floating point number that can be represented by a digital machine. Now let us show that $\log n C_{ub} / \epsilon_{ps}$ is polynomial in $I_{e_1}$.

According to the definition of the fuel-time function $c_{e_1}()$ in (2), we get

\begin{align*}
\log \left( \frac{C_{ub}}{\epsilon_{ps}} \right) &= \log \left( \frac{c_{e_1}(t_{eb}^l)}{\epsilon_{ps}} \right) \\
&= \log \left( \frac{t_{eb}^l \cdot f_{e_1}(D_{e_1}^{ub} / t_{eb}^l)}{\epsilon_{ps}} \right) = \log \left( \frac{D_{e_1}^{ub} / t_{eb}^l \cdot f_{e_1}(R_{e_1}^{ub})}{\epsilon_{ps}} \right) \\
&= \log \left( \frac{D_{e_1}^{ub}}{R_{e_1}^{ub}} \right) + \log \left( \frac{f_{e_1}(R_{e_1}^{ub})}{\epsilon_{ps}} \right) = \log \left( \frac{D_{e_1}}{R_{e_1}^{ub}} \right) - \log \left( \frac{R_{e_1}^{ub}}{\epsilon_{ps}} \right) + \log \left( \frac{f_{e_1}(R_{e_1}^{ub})}{\epsilon_{ps}} \right) \\
&\leq \log \left( I_{e_1} + \log \left( \frac{f_{e_1}(R_{e_1}^{ub})}{\epsilon_{ps}} \right) \right) \\
&\leq \log \left( I_{e_1} + \log \left( \frac{a_1 (R_{e_1}^{ub})^{k_1} + \cdots + a_g (R_{e_1}^{ub})^{k_g}}{\epsilon_{ps}} \right) \right) \\
&\leq \log \left( I_{e_1} + \log \left( \frac{a_i (R_{e_1}^{ub})^{k_i}}{\epsilon_{ps}} \right) \right) \\
&= \log \left( I_{e_1} + \log q + \log \left( \frac{a_i}{\epsilon_{ps}} \right) + \log \left( \frac{R_{e_1}^{ub}}{\epsilon_{ps}} \right) \cdot \epsilon_{ps}^{k_i} \right),
\end{align*}

(Since $\log \epsilon_{ps} < 0$)

which is thus polynomial in $I_{e_1}$.

Then

\begin{align*}
\log \log n C_{ub} / \epsilon_{ps} &= \log \log \frac{C_{ub}}{\epsilon_{ps}} \leq \log \log n \epsilon_{ps} \\
&= \log \left( \log n + \log \frac{C_{ub}}{C_{lb}} \right) \leq 2 \max \left\{ \log \log n, \log \frac{C_{ub}}{C_{lb}} \right\} \\
&= \max \{ O(\log \log n), O(I_{e_1}) \}
\end{align*}

which is polynomial in the input size of PASO because both $O(\log \log n)$ and $O(I_{e_1})$ are polynomial in the input size of PASO. We thus prove the second part of this theorem.

The proof is completed.

**APPENDIX G**

**PROOF OF LEMMA 6**

Define function $h(t_e) = c_{e}(t_e) + \lambda t_e$. Then we can get the first derivative as

$$h'(t_e) = c'_e(t_e) + \lambda.$$  \hspace{1cm} (37)

Since $c_{e}(t_e)$ is a strictly convex and strict decreasing function, we know that $c'_e(t_e)$ (and also $h'(t_e)$) is a strictly increasing function and $c'_e(t_e) < 0$ at interval $[t_{eb}^l, t_{eb}^{ub}]$. We then consider the following three cases.

**Case 1:** If $0 \leq \lambda < -c'_e(t_e^{ub})$, we get that $c'_e(t_e^{ub}) + \lambda < 0$ and thus

$$h'(t_e) \leq h'(t_e^{ub}) < 0, \forall, t \in [t_{eb}^l, t_{eb}^{ub}].$$

(38)

This shows that $h(t_e)$ is strictly decreasing at $[t_{eb}^l, t_{eb}^{ub}]$ and the minimal value is attained at $t_{eb}^* = t_{eb}^{lb}$.

**Case 2:** If $-c'_e(t_e^{ub}) \leq \lambda \leq -c'_e(t_e^{lb})$, then we can get that $c_{e}^{-1}(\lambda - t_{eb}^{lb}) \in [t_{eb}^{lb}, t_{eb}^{ub}]$. Clearly, the monotonic increasing property of $h'(t_e)$ implies that $h'(t_e) < 0$ at $[t_{eb}^{lb}, c_{e}^{-1}(\lambda - t_{eb}^{lb})]$ and $h'(t_e) > 0$ at $(c_{e}^{-1}(\lambda - t_{eb}^{lb}), t_{eb}^{ub}]$. This means that the minimal value is attained at $t_{eb}^* = c_{e}^{-1}(\lambda - t_{eb}^{lb})$.

**Case 3:** If $\lambda > -c'_e(t_e^{lb})$, we get that $c'_e(t_e^{lb}) + \lambda > 0$ and thus

$$h'(t_e) \geq h'(t_e^{lb}) > 0, \forall, t \in [t_{eb}^{lb}, t_{eb}^{ub}].$$

(39)

This shows that $h(t_e)$ is strictly increasing at $[t_{eb}^{lb}, t_{eb}^{ub}]$ and the minimal value is attained at $t_{eb}^* = t_{eb}^{ub}$.

The proof is completed.
APPENDIX H
PROOF OF THEOREM 3
Let us consider any two \( \lambda_1, \lambda_2 \) with \( 0 \leq \lambda_1 < \lambda_2 \). We need to prove \( \delta(\lambda_1) \geq \delta(\lambda_2) \). Suppose that the optimal path at \( \lambda_1 \) is \( p^*(\lambda_1) = p_1 \) and the optimal path at \( \lambda_2 \) is \( p^*(\lambda_2) = p_2 \).

For any path \( p \) and any \( \lambda \geq 0 \), we denote its (optimal) generalized path cost as
\[
W_p(\lambda) \triangleq \sum_{\lambda \in p} w_e(\lambda) = \sum_{\lambda \in p} [c_e(t^*_e(\lambda)) + \lambda t^*_e(\lambda)],
\]
and denote its corresponding path fuel cost as
\[
C_p(\lambda) \triangleq \sum_{\lambda \in p} c_e(t^*_e(\lambda)).
\]
and denote its corresponding path delay
\[
T_p(\lambda) \triangleq \sum_{\lambda \in p} t^*_e(\lambda).
\]
Clearly, we have \( W_p(\lambda) = C_p(\lambda) + \lambda T_p(\lambda) \).

Based on such notations, we have \( \delta(\lambda_1) = T_{p_1}(\lambda_1) \) and \( \delta(\lambda_2) = T_{p_2}(\lambda_2) \), and we need to prove \( T_{p_1}(\lambda_1) \geq T_{p_2}(\lambda_2) \).

When \( \lambda = \lambda_1 \), the optimal path is \( p_1 \), which means that
\[
W_{p_1}(\lambda_1) = C_{p_1}(\lambda_1) + \lambda_1 T_{p_1}(\lambda_1) \
\leq W_{p_2}(\lambda_1) = C_{p_2}(\lambda_1) + \lambda_1 T_{p_2}(\lambda_1) \tag{43}
\]
Similarly, when \( \lambda = \lambda_2 \), the optimal path is \( p_2 \), which means that
\[
W_{p_2}(\lambda_2) = C_{p_2}(\lambda_2) + \lambda_2 T_{p_2}(\lambda_2) \leq W_{p_1}(\lambda_2) = C_{p_1}(\lambda_2) + \lambda_2 T_{p_1}(\lambda_2) \tag{44}
\]
Now we will use the fact that \( t^*_e(\lambda) \) minimizes \( w_e(\lambda) \), as defined in (13). Since both \( t^*_e(\lambda_1) \) and \( t^*_e(\lambda_2) \) are feasible, i.e., in the interval \([t^b_e, t^u_e] \), we get that
\[
W_{p_2}(\lambda_1) = C_{p_2}(\lambda_1) + \lambda_1 T_{p_2}(\lambda_1) \
= \sum_{\lambda \in p_2} (c_e(t^*_e(\lambda_1)) + \lambda_1 t^*_e(\lambda_1)) \
= \sum_{\lambda \in p_2} \min_{t_e \in [t^b_e, t^u_e]} (c_e(t_e) + \lambda_1 t_e) \
\leq \sum_{\lambda \in p_2} (c_e(t^*_e(\lambda_2)) + \lambda_1 t^*_e(\lambda_2)) \
= C_{p_2}(\lambda_2) + \lambda_1 T_{p_2}(\lambda_2). \tag{45}
\]
Similarly, we have
\[
W_{p_1}(\lambda_2) = C_{p_1}(\lambda_2) + \lambda_2 T_{p_1}(\lambda_2) \leq C_{p_1}(\lambda_1) + \lambda_2 T_{p_1}(\lambda_1). \tag{46}
\]

Inserting (45) into (43), we get that
\[
C_{p_1}(\lambda_1) + \lambda_1 T_{p_1}(\lambda_1) \leq C_{p_2}(\lambda_2) + \lambda_1 T_{p_2}(\lambda_2),
\]
which implies that
\[
\lambda_1 [T_{p_1}(\lambda_1) - T_{p_2}(\lambda_2)] \leq C_{p_2}(\lambda_2) - C_{p_1}(\lambda_1). \tag{47}
\]
Similarly, inserting (46) into (44), we get that
\[
C_{p_2}(\lambda_2) + \lambda_2 T_{p_2}(\lambda_2) \leq C_{p_1}(\lambda_1) + \lambda_2 T_{p_1}(\lambda_1),
\]
which implies that
\[
-\lambda_2 [T_{p_1}(\lambda_1) - T_{p_2}(\lambda_2)] \leq C_{p_1}(\lambda_1) - C_{p_2}(\lambda_2). \tag{48}
\]
Summing (47) and (48), we get that
\[
(\lambda_1 - \lambda_2) [T_{p_1}(\lambda_1) - T_{p_2}(\lambda_2)] \leq 0. \tag{49}
\]
Since we assume that \( \lambda_1 < \lambda_2 \), we must have
\[
T_{p_1}(\lambda_1) \geq T_{p_2}(\lambda_2). \tag{50}
\]
The proof is completed.

APPENDIX I
PROOF OF THEOREM 4
At the point \( \lambda_0 \), the dual function has value
\[
D(\lambda_0) = -\lambda_0 T + \min_{x \in X} \sum_{c_e \in E} x_c \cdot \min_{t_e \in [t^b_e, t^u_e]} (c_e(t_e) + \lambda_0 t_e).
\]
\[
= -\lambda_0 T + \min_{x \in X} \sum_{c_e \in E} x_c \cdot \sum_{t_e \in [t^b_e, t^u_e]} (c_e(t_e) + \lambda_0 t_e).
\]
\[
= -\lambda_0 T + \sum_{c_e \in E} \sum_{t_e \in [t^b_e, t^u_e]} c_e(t_e) + \lambda_0 \sum_{t_e \in [t^b_e, t^u_e]} t_e.
\]
\[
= -\lambda_0 T + \lambda_0 \delta(\lambda_0) + \sum_{c_e \in E} \sum_{t_e \in [t^b_e, t^u_e]} c_e(t_e).
\]
\[
= \sum_{c_e \in E} c_e(t^*_e(\lambda_0)). \tag{51}
\]
On one hand, we know that any dual function value will be a lower bound of \( \text{OPT} \) according to the weak duality. Thus,
\[
D(\lambda_0) \leq \text{OPT}. \tag{52}
\]
On the other hand, we know that \( p^*(\lambda_0) \) is a feasible path and \( \{t^*_e(\lambda_0), e \in p^*(\lambda_0)\} \) satisfies
\[
\sum_{e \in p^*(\lambda_0)} t^*_e(\lambda_0) = T. \tag{53}
\]
Here \( p^*(\lambda_0) \) and \( \{t^*_e(\lambda_0), e \in p^*(\lambda_0)\} \) is a feasible solution to \( \text{PASO} \) with the objective value \( \sum_{e \in p^*(\lambda_0)} c_e(t^*_e(\lambda_0)) = D(\lambda_0) \), which is an upper bound of \( \text{OPT} \), i.e.,
\[
D(\lambda_0) \geq \text{OPT}. \tag{54}
\]
Eq. (52) and (54) conclude that \( D(\lambda_0) = \text{OPT} \), and \( p^*(\lambda_0) \) and \( \{t^*_e(\lambda_0), e \in p^*(\lambda_0)\} \) is an optimal solution to \( \text{PASO} \).

The proof is completed.

APPENDIX J
PROOF OF THEOREM 5
First, if we let total travel delay be \( T' = \sum_{c_e \in E} t^*_e(\lambda_L) > T \), we get a relaxed version of \( \text{PASO} \). According to Theorem 4, we know that \( \text{LB} = \sum_{c_e \in E} c_e(t^*_e(\lambda_L)) \) is the optimal solution of the relaxed version, and thus we have \( \text{LB} \leq \text{OPT} \).
Second, since \( \sum_{e \in p^*(\lambda U)} t_e^*(\lambda U) < T \), we know that

\( p^*(\lambda U) \) and \( \{t_e^*(\lambda U) : e \in p^*(\lambda U)\} \) is a feasible solution
to \( \text{PASO} \). Thus, \( UB = \sum_{e \in p^*(\lambda L)} c_e(t_e^*(\lambda U)) \geq OPT \).

The proof is completed.